2021 James S. Rickards Fall Invitational

For all questions, answer choice (E) NOTA means that none of the given answers is correct. Good Luck!

- 1. Minnie Mouse loves polka dots. One of her favorite dresses has polka dots whose radius is equal to the distance between the points (-13, 29) and (4, 18). Find the area of the polka dots on Minnie Mouse's favorite dress in terms of π .
 - (A) 410π (B) 400π (C) 202π (D) 305π (E) NOTA
- 2. Given the following system of equations:

$$6x - 18z = 66$$

$$14x - 8y - 2z = -30$$

$$8x + 12y + 10z = -12$$

3. Anya could use some help on her math homework. Being the generous person you are, you decide to help her out by solving this problem: Find the remainder of $\frac{8x^4 + 6x^3 - 3x^2 + 4x + 9}{x+4}$. Assuming you solved the problem correctly, what answer did you get? (A) 1600 (B) 1509 (C) 1501 (D) -1641 (E) NOTA

4. The expression $343a^3 + 27b^6$ can be written as a product of $7a + 3b^2$ and another factor. Find the other factor. (A) $49a^2 - 42ab^2 + 9b^4$ (B) $49a^2 - 21ab^2 + 9b^4$ (C) $49a^2 + 9b^6$ (D) $49a^2 + 21ab^2 + 9b^4$ (E) NOTA

5. Nihar likes to think rationally. Make him proud by rationalizing the denominator of the fraction $\frac{7}{\sqrt{2} + \sqrt{3} - \sqrt{5}}$ (A) $\frac{7(\sqrt{30} + 2\sqrt{3} + 3\sqrt{2})}{12}$ (B) $\frac{7(\sqrt{2} - \sqrt{3})}{6}$ (C) $\frac{7(\sqrt{6} + \sqrt{30})}{12}$ (D) $\frac{7(\sqrt{3} + 2\sqrt{2} - \sqrt{30})}{6}$ (E) NOTA

- 6. Calculate the product of the number of integers and the number of natural numbers in the following list: -9, 0.75, 2, 0, 11/3, -1, -29, 3, 1, -8.
 (A) 15
 (B) 18
 (C) 24
 (D) 14
 (E) NOTA
- 7. Gabbi and Navya love sour patch kids. When Gabbi is snacking alone, she can finish a whole bag of the candy in 14 minutes whereas Navya takes 21 minutes. One day, Gabbi and Navya decide to share a bag of sour patch kids. Sadly, they are only able to enjoy the snack for 6 minutes before they get called away to look at a cute frog. What fraction of the bag is left after Gabbi and Navya stop eating?
 - (A) $\frac{37}{42}$ (B) $\frac{5}{42}$ (C) $\frac{5}{7}$ (D) $\frac{2}{7}$ (E) NOTA
- 8. In how many distinct ways can the letters in the word ACKERMAN be arranged so that the E and K are not next to each other?
 - (A) 17640 (B) 35280 (C) 5040 (D) 8820 (E) NOTA
- 9. The number of animes Riya can watch in a week is equal to the sum of the following infinite series:

$$2, \frac{4}{3}, \frac{8}{9}, \frac{16}{27} \dots$$

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How many animes can Riya watch in a week?						
(A) 4	(B) 6	(C) 8	(D) 3	(E) NOTA		

- 10. Geetika and Navya have gotten lost in Switzerland and are trying to find their way back to one another. Geetika is travelling down the path $y = 3x^2 15x + 21$. Navya is travelling down the line 6x y = -3. Find the product of the ordinates of the points at which their paths will intersect.
 - (A) 0 (B) 6 (C) 351 (D) 54 (E) NOTA

11. If the equation $y = -18x^2 - 72x - 32$ represents a parabola with vertex (a, b), what is 2ab?

- (A) $\frac{80}{9}$ (B) 160 (C) -128 (D) -80 (E) NOTA
- 12. If $f(x) = \frac{3x^3 10}{2}$, find the value of $f^{-1}(8.5)$. (A) $\sqrt{9}$ (B) $3\sqrt[3]{3}$ (C) 3 (D) $\sqrt[3]{9}$ (E) NOTA

13. Manjari is planning to buy tickets for the latest BTS concert but is struggling to choose seats for her and her friends. Based on her calculations, there are 10,626 ways for her to choose seats. If there are 24 seats available, how many friends is Manjari planning to take to the concert?

(A) 5 (B) 3 (C) 4 (D) 2 (E) NOTA

14. At Kaigen Middle School, students must choose to participate in sword-making, dueling, or both. Of the 40 people who participate in dueling, 55% also participate in sword-making. If 31 people participate in sword-making, how many of them did not also join dueling?

(A) 9 (B) 29 (C) 11 (D) 20 (E) NOTA

15. The three Kevins are fighting for supremacy. They decide that whoever can answer the following question correctly shall be named the Kevin of all Kevins. The question reads: Find the domain of the function $f(x) = \frac{x^3 - 9}{(2x^2 + 4x - 16)^{1/2}}$. What is the correct answer? (A) $(-\infty, -4] \cup (2, \infty)$ (B) (-4, 2) (C) $(-\infty, -4] \cup [2, \infty)$ (D) $(-\infty, \infty)$ (E) NOTA

- 16. If $g(x) = \sqrt{x^3 + 16}$ and $f(x) = x^2 11$, find $f^2(g(3))$. (A) 184 (B) 1024 (C) 32 (D) 1013 (E) NOTA
- 17. A certain quadrilateral ABCD has vertices A(3, 8), B(7, 6), C(5, -2), and D(-1, 4) on the Cartesian plane. Find the perimeter of the quadrilateral formed by connecting the midpoints of the sides of quadrilateral ABCD.

(A)
$$12 + 2\sqrt{29}$$
 (B) $\sqrt{2} + \sqrt{17} + 2\sqrt{26}$ (C) $2\sqrt{17} + 2\sqrt{26}$ (D) $6 + 2\sqrt{26}$ (E) NOTA

18. Solve the equation $\frac{1}{9^2} = 3^{2x-7}$.

(A) $\frac{3}{2}$ (B) $\frac{11}{2}$ (C) $\frac{-3}{2}$ (D) $\frac{5}{2}$ (E) NOTA

19. Jason is one step away from unlocking the secrets of the universe. This final step, however, requires that he solve a math problem that has Jason scratching his head. Help Jason unlock the secrets of the universe by solving this problem: Find all values of x that satisfy the inequality $5x^2 - 10x \ge 3(x-4) + 6$.

(A)
$$(\frac{3}{5}, 2)$$
 (B) $(-\infty, \frac{3}{5}) \cup (2, \infty)$ (C) $(-\infty, \frac{3}{5}] \cup [2, \infty)$ (D) $(-\infty, \frac{3}{5}] \cup [\frac{3}{5}, 2] \cup [2, \infty)$ (E) NOTA

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20.	The expression $\sqrt{48-64}$	$\overline{4i}$ can be simplified into t	he form $a + bi$ where a is	s positive. Find $a + b$.			
	(A) 12	(B) -4	(C) -12	(D) 8	(E) NOTA		

21. Navya enjoys her beauty sleep. The amount of sleep she gets is directly proportional to the number of Cheetos she consumes and inversely proportional to the number of hours of Netflix she watches on any given day. When Navya watches 4 hours of Netflix and eats 11 Cheetos, she gets 7 hours of sleep. Today, she eats 22 Cheetos and gets 10 hours of sleep. How many hours of Netflix did she watch, to the nearest tenths place?
(A) 2.9
(B) 5.6
(C) 1.2
(D) 3.4
(E) NOTA

22. Simplify the expression
$$\sqrt{8 + 2\sqrt{7}} + \sqrt{8 - 2\sqrt{7}}$$
.
(A) $\sqrt{8} - \sqrt{7}$ (B) 6 (C) $\sqrt{7}$ (D) 4 (E) NOTA

23. You are dying to learn the sacred art of firebending from the one and only, Zuko. However, Zuko will only agree to teach you firebending if you answer this problem correctly: If f(x) = x² − 4x + 15 and g(x) = 3x³ − 7x² + 2x − 9, what is the sum of the degree and constant term of f(x) ⋅ g(x)?

$$(A) -129 (B) -132 (C) -135 (D) -130 (E) NOTA$$

24. Simplify:

$$\frac{1}{7-4\sqrt{3}} - \frac{1}{4\sqrt{3}-\sqrt{47}} + \frac{1}{\sqrt{47}-\sqrt{46}} - \frac{1}{\sqrt{46}-3\sqrt{5}}$$
(A) $7 + 3\sqrt{5}$ (B) $3\sqrt{5}$ (C) $7 - 3\sqrt{5}$ (D) 1 (E) NOTA

- 25. Velan's favorite angle is equal to the measure of the interior angle of a regular nonagon. Find the supplement of the complement of half of Velan's favorite angle.
 - (A) 70 (B) 130 (C) 160 (D) 110 (E) NOTA
- 26. Given

$$f(x) = \begin{cases} 6x - 11, \text{ if } x^2 < 25\\ 3x^2 - 14x - 5, \text{ if } -8 < x \le -5\\ -2(\frac{x}{3}), \text{ if } x < -9 \end{cases}$$

Find
$$f(3) - f(-6) + f(f(-4))$$
.
(A) $\frac{-470}{3}$ (B) $\frac{29}{3}$ (C) $\frac{610}{3}$ (D) $\frac{652}{3}$ (E) NOTA

27. Gabbi is playing soccer with some friends. At one point, the height, in feet, of the ball at t seconds after Gabbi kicks it is given by $-5t^2 + 34t - 24$. What is the greatest height the ball reaches? All answer choices are in terms of feet.

(A)
$$\frac{169}{5}$$
 (B) 24 (C) $\frac{17}{5}$ (D) $\frac{125}{6}$ (E) NOTA

28. Solve ||4x - 9| + 5| = 17.

(A)
$$\frac{11}{2}, \frac{-7}{2}$$
 (B) $\frac{-13}{4}, \frac{21}{4}$ (C) $\frac{13}{4}, \frac{21}{4}$ (D) $\frac{21}{4}$ (E) NOTA

29. Find the equation of the perpendicular bisector of the line segment with endpoints (-9, 14) and (5, -12).

(A)
$$y = \frac{7}{13}x - \frac{27}{13}$$
 (B) $y = \frac{-7}{13}x - \frac{1}{13}$ (C) $y = \frac{-13}{7}x + \frac{27}{7}$ (D) $y = \frac{7}{13}x + \frac{27}{13}$ (E) NOTA

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30. You've made it to the end! Give yourself a pat on the back. In celebration, this question will be fairly straightforward. What is the sum of the distinct factors of 2021?

$(\mathbf{A}$) 2113 ((B) 2111	(C) 2022	(D) 2064	(E) NOTA
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